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# A Unified Spinorial Superfield Treatment of the Higher Superspin Superfield Formalism

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## ABSTRACT

We discuss the higher superspin superfield formalism of Kuzenko et. al. from the basis of a unified treatment of a spinorial superfield prepotential, its action and a restricted set of gauge transformations. We recover previous results as distinct limits of this unified treatment and give the first derivations of the complete set of Bianchi identities associated with these equations.

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# 1 Introduction

In 1993, two theories for higher massless superspins (one each for the cases of integer and half odd superspin) were developed and established in the literature [1, 2]. An interesting observation is that in both theories, there exists a spinorial superfield (physical superfield for the integer case, auxiliary superfield for the half odd case). This observation motivates us to look for a theory of a spinorial superfield with some free parameters, that in a way unifies the two previously known theories as a first step to investigate more general possibilities. More specifically, for certain values of the free parameters we recover the integer case and for another point in the parameter space we recover the half odd case. In the following, we will also derive the field strength superfields expressed as funcitons of the appropriate prepotentials as well as the explicit form of their Bianchi identities.

# 2 A Proposed Action and Symmetry

The starting point would be to find the most general action for a free massless spinorial superfield $\Psi_{\alpha(s)\dot{\alpha}(s-1)}$. Furthermore since this object will be the physical superfield in the case of integer superspin, this means that the highest spin included in the supermultiplet is a fermion. Therefore the $\theta\bar{\theta}$ component of $\Psi$ has mass dimensions 3/2, which means that $\Psi$ itself has mass dimensions 1/2. For a 4D, $\mathcal{N} = 1$ supersymmetric theory, the measure of integration over superspace $d^8z$ has mass dimensions -2, so in order our action (quadratic in $\Psi$) to be dimensionless we need 2 spinorial derivatives.

The most general action for such a spinorial superfield is

$$S = \int d^8z \Big\{ c_1 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \mathrm{D}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c_2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\ + a_1 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}_{\alpha_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} + a_2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \mathrm{D}_{\alpha_s} \bar{\mathrm{D}}^{\dot{\alpha}_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \Big\} \tag{1}$$

where the parameters $a_i \in \mathbb{R}$ and $c_i \in \mathbb{C}$ in complete generality.

If this action is to describe a massless supermultiplet, it should have some gauge symmetry. We thus demand that this action is invariant under a specific gauge transformation. The way to choose this transformation is to look for symmetries that respect the highest superspin projector operator $\Pi$ [3]

$$(\Pi\Psi)_{\alpha(2s-1)} \propto \mathrm{D}^{\alpha_{2s}} \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_{2s}} \partial^{\dot{\alpha}_1}{}_{\alpha_{2s-1}} \dots \partial^{\dot{\alpha}_{s-1}}{}_{\alpha_{s+1}} \Psi_{\alpha(s))\dot{\alpha}(s-1)} \tag{2}$$

From this it is obvious there are four distinct possible gauge transformation laws of $\Psi$ that preserve the highest spin projection operator. These individually take the forms

$$\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} = \frac{1}{s!}\mathrm{D}_{(\alpha_s}K_{\alpha(s-1))\dot{\alpha}(s-1)} \tag{3a}$$

$$\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} = \frac{1}{(s-1)!}\bar{\mathrm{D}}_{(\dot{\alpha}_{s-1}}\Lambda_{\alpha(s)\dot{\alpha}(s-2))} \tag{3b}$$

$$\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} = \mathrm{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)} \tag{3c}$$

$$\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} = \bar{\mathrm{D}}^2 U_{\alpha(s)\dot{\alpha}(s-1)} \tag{3d}$$

and the the most general gauge transformation law can include linear combinations of these. For the considerations of this note we pick the following gauge transformation[3]

$$\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} = \mathrm{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}^2 U_{\alpha(s)\dot{\alpha}(s-1)} \tag{4}$$

where $[L] = [U] = -1/2$.

Under this transformation, the different terms in the action transforms as following:

$$\begin{aligned}
\mathrm{D}^2\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} &= \mathrm{D}^2\bar{\mathrm{D}}^2 U_{\alpha(s)\dot{\alpha}(s-1)} \\
\bar{\mathrm{D}}^2\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} &= \bar{\mathrm{D}}^2\mathrm{D}^2 L_{\alpha(s))\dot{\alpha}(s-1)} \\
\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}_{\dot{\alpha}_s}\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} &= \mathrm{D}^{\alpha_s}\bar{\mathrm{D}}_{\dot{\alpha}_s}\mathrm{D}^2 L_{\alpha(s))\dot{\alpha}(s-1)} \\
\bar{\mathrm{D}}_{\dot{\alpha}_s}\mathrm{D}^{\alpha_s}\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} &= \bar{\mathrm{D}}_{\dot{\alpha}_s}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^2 U_{\alpha(s)\dot{\alpha}(s-1)} \quad .
\end{aligned} \tag{5}$$

Thus making the change of the superfield according to

$$\Psi_{\alpha(s)\dot{\alpha}(s-1)} \to \Psi_{\alpha(s)\dot{\alpha}(s-1)} + \delta\Psi_{\alpha(s)\dot{\alpha}(s-1)}$$

leads to a change in the action as:

$$\begin{aligned}
\delta S = \int d^8z\, &\left\{\frac{2s}{s+1}c_1\bar{\mathrm{D}}^{\dot{\alpha}_s}\mathrm{D}^2\Psi^{\alpha(s)\dot{\alpha}(s-1)} + a_2\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^2\bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)}\right\}\frac{1}{s!}\bar{\mathrm{D}}_{(\dot{\alpha}_s}U_{\alpha(s)\dot{\alpha}(s-1))} \\
&+\left\{\frac{2s}{s+1}c_1\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^2\bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} + a_2\bar{\mathrm{D}}^{\dot{\alpha}_s}\mathrm{D}^2\Psi^{\alpha(s)\dot{\alpha}(s-1)}\right\}\frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{U}_{\alpha(s-1))\dot{\alpha}(s)} \\
&+\left\{-2c_2\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^2\Psi^{\alpha(s)\dot{\alpha}(s-1)} + a_1\bar{\mathrm{D}}_{\dot{\alpha}_s}\mathrm{D}^2\bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)}\right\}\mathrm{D}^{\beta}L_{\beta\alpha(s-1)\dot{\alpha}(s-1)} \\
&+\left\{-2c_2\bar{\mathrm{D}}_{\dot{\alpha}_s}\mathrm{D}^2\bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} + a_1\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^2\Psi^{\alpha(s)\dot{\alpha}(s-1)}\right\}\bar{\mathrm{D}}^{\dot{\beta}}\bar{L}_{\alpha(s-1)\dot{\alpha}(s-1)\dot{\beta}} \quad .
\end{aligned} \tag{6}$$

Upon choosing (in order to construct a minimal theory)

$$\frac{2s}{s+1}c_1 = -a_2 \tag{7a}$$

$$2c_2 = -a_1 \quad , \tag{7b}$$

[3]The general case is been discussed in [4] and [5]

the change of the action is

$$
\begin{aligned}
\delta S = \int d^8 z \Big\{ & - a_2 \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} + a_2 \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \Big\} \Big[ \frac{1}{s!} \bar{\mathrm{D}}_{(\dot{\alpha}_s} U_{\alpha(s)\dot{\alpha}(s-1))} \Big] \\
& + \Big\{ a_2 \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} - a_2 \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \Big\} \Big[ \frac{1}{s!} \mathrm{D}_{(\alpha_s} \bar{U}_{\alpha(s-1))\dot{\alpha}(s)} \Big] \\
& + \Big\{ a_1 \mathrm{D}_{\alpha_s} \bar{\mathrm{D}}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} + a_1 \bar{\mathrm{D}}_{\dot{\alpha}_s} \mathrm{D}^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \Big\} \Big[ \mathrm{D}^{\beta} L_{\beta\alpha(s-1)\dot{\alpha}(s-1)} \Big] \\
& + \Big\{ a_1 \mathrm{D}_{\alpha_s} \bar{\mathrm{D}}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} + a_1 \bar{\mathrm{D}}_{\dot{\alpha}_s} \mathrm{D}^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \Big\} \Big[ \bar{\mathrm{D}}^{\dot{\beta}} \bar{L}_{\alpha(s-1)\dot{\alpha}(s-1)\dot{\beta}} \Big] \quad .
\end{aligned}
\tag{8}
$$

# 3 Compensators and Bianchi Identities

To compensate for the change of the action, we introduce two real superfields $H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)}$ and $H^{(2)}_{\alpha(s)\dot{\alpha}(s)}$ ($[H^{(1)}] = [H^{(2)}] = 0$) which transform:

$$
\delta H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} = \mathrm{D}^{\alpha_s} L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}^{\dot{\alpha}_s} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)} \tag{9}
$$

$$
\delta H^{(2)}_{\alpha(s)\dot{\alpha}(s)} = \frac{1}{s!} \bar{\mathrm{D}}_{(\dot{\alpha}_s} U_{\alpha(s)\dot{\alpha}(s-1))} - \frac{1}{s!} \mathrm{D}_{(\alpha_s} \bar{U}_{\alpha(s-1))\dot{\alpha}(s)} \tag{10}
$$

and add the following terms in the action:

1.) a cross term (interaction between the compensators and $\Psi$)

$$
\begin{aligned}
S_c = \int d^8 z a_2 & \Big\{ \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} - \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \Big\} H^{(2)}_{\alpha(s)\dot{\alpha}(s)} \\
& - a_1 \Big\{ \mathrm{D}_{\alpha_s} \bar{\mathrm{D}}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}_{\dot{\alpha}_s} \mathrm{D}^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \Big\} H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} \quad ,
\end{aligned}
\tag{11}
$$

2.) a kinetic energy term for $H^{(1)}$ (the most general action for a free massless superfield with mass dimensions zero)

$$
\begin{aligned}
S_{k_1} = \int d^8 z \Big\{ & A_1 H^{(1)\alpha(s-1)\dot{\alpha}(s-1)} D^{\gamma} \bar{D}^2 D_{\gamma} H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} \\
& + A_2 H^{(1)\alpha(s-1)\dot{\alpha}(s-1)} \Box H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} \\
& + A_3 H^{(1)\beta\alpha(s-2)\dot{\beta}\dot{\alpha}(s-2)} [D_{\beta}, \bar{D}_{\dot{\beta}}][D^{\gamma}, \bar{D}^{\dot{\gamma}}] H^{(1)}_{\gamma\alpha(s-2)\dot{\gamma}\dot{\alpha}(s-2)} \\
& + A_4 H^{(1)\beta\alpha(s-2)\dot{\beta}\dot{\alpha}(s-2)} \partial_{\beta\dot{\beta}} \partial^{\gamma\dot{\gamma}} H^{(1)}_{\gamma\alpha(s-2)\dot{\gamma}\dot{\alpha}(s-2)} \Big\}
\end{aligned}
\tag{12}
$$

and,

3.) a kinetic energy term for $H^{(2)}$

$$
\begin{aligned}
S_{k_2} = \int d^8 z \Big\{ & B_1 H^{(2)\alpha(s)\dot\alpha(s)} D^\gamma \bar D^2 D_\gamma H^{(2)}_{\alpha(s)\dot\alpha(s)} \\
& + B_2 H^{(2)\alpha(s)\dot\alpha(s)} \Box H^{(2)}_{\alpha(s)\dot\alpha(s)} \\
& + B_3 H^{(2)\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} [D_\beta, \bar D_{\dot\beta}][D^\gamma, \bar D^{\dot\gamma}] H^{(2)}_{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-1)} \\
& + B_4 H^{(2)\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} \partial_{\beta\dot\beta} \partial^{\gamma\dot\gamma} H^{(2)}_{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-1)} \Big\} \quad .
\end{aligned}
\tag{13}
$$

Therefore the complete action is

$$
\mathcal{S} = S + S_c + S_{k_1} + S_{k_2} \quad . \tag{14}
$$

Based on this action we calculate the variations with respect each superfield:

$$
\begin{aligned}
\mathcal{T}_{\alpha(s)\dot\alpha(s-1)} = & \frac{\delta \mathcal{S}}{\delta \Psi^{\alpha(s)\dot\alpha(s-1)}} \\
= & - \frac{s+1}{s} a_2 \mathrm{D}^2 \Psi_{\alpha(s)\dot\alpha(s-1)} - a_1 \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot\alpha(s-1)} \\
& + \frac{1}{s!} a_1 \bar{\mathrm{D}}^{\dot\alpha_s} \mathrm{D}_{(\alpha_s} \bar\Psi_{\alpha(s-1))\dot\alpha(s)} + \frac{1}{s!} a_2 \mathrm{D}_{\alpha_s} \bar{\mathrm{D}}^{\dot\alpha_s} \bar\Psi_{\alpha(s-1)\dot\alpha(s)} \\
& + a_2 \mathrm{D}^2 \bar{\mathrm{D}}^{\dot\alpha_s} H^{(2)}_{\alpha(s)\dot\alpha(s)} - \frac{1}{s!} a_1 \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_s} H^{(1)}_{\alpha(s-1))\dot\alpha(s-1)}
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
\bar{\mathcal{T}}_{\alpha(s-1)\dot\alpha(s)} = & - \frac{s+1}{s} a_2 \bar{\mathrm{D}}^2 \bar\Psi_{\alpha(s-1)\dot\alpha(s)} - a_1 \mathrm{D}^2 \bar\Psi_{\alpha(s-1)\dot\alpha(s)} \\
& + \frac{1}{s!} a_1 \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}_{(\dot\alpha_s} \Psi_{\alpha(s)\dot\alpha(s-1))} + \frac{1}{s!} a_2 \bar{\mathrm{D}}_{(\dot\alpha_s} \mathrm{D}^{\alpha_s} \Psi_{\alpha(s)\dot\alpha(s-1))} \\
& - a_2 \bar{\mathrm{D}}^2 \mathrm{D}^{\alpha_s} H^{(2)}_{\alpha(s)\dot\alpha(s)} - \frac{1}{s!} a_1 \mathrm{D}^2 \bar{\mathrm{D}}_{(\dot\alpha_s} H^{(1)}_{\alpha(s-1)\dot\alpha(s-1))}
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\mathcal{G}^{(1)}_{\alpha(s-1)\dot\alpha(s-1)} = & \frac{\delta \mathcal{S}}{\delta H^{(1)\alpha(s-1)\dot\alpha(s-1)}} \\
= & + a_1 D^{\alpha_s} \bar D^2 \Psi_{\alpha(s)\dot\alpha(s-1)} + a_1 \bar D^{\dot\alpha_s} D^2 \bar\Psi_{\alpha(s-1)\dot\alpha(s)} \\
& + 2A_1 D^\gamma \bar D^2 D_\gamma H^{(1)}_{\alpha(s-1)\dot\alpha(s-1)} + 2A_2 \Box H^{(1)}_{\alpha(s-1)\dot\alpha(s-1)} \\
& + \frac{2}{(s-1)!^2} A_3 [D_{(\alpha_{s-1}}, \bar D_{(\dot\alpha_{s-1}}][D^\gamma, \bar D^{\dot\gamma}] H^{(1)}_{\gamma\alpha(s-2))\dot\gamma\dot\alpha(s-2))} \\
& + \frac{2}{(s-1)!^2} A_4 \partial_{(\alpha_{s-1}(\dot\alpha_{s-1}} \partial^{\gamma\dot\gamma} H^{(1)}_{\gamma\alpha(s-2))\dot\gamma\dot\alpha(s-2))}
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\mathcal{G}^{(2)}_{\alpha(s)\dot\alpha(s)} =& \frac{\delta\mathcal{S}}{\delta H^{(2)\alpha(s)\dot\alpha(s)}} \\
=&+\frac{1}{s!}a_2\bar{D}_{(\dot\alpha_s}D^2\Psi_{\alpha(s)\dot\alpha(s-1))}-\frac{1}{s!}a_2D_{(\alpha_s}\bar{D}^2\bar\Psi_{\alpha(s-1))\dot\alpha(s)} \\
&+2B_1D^\gamma\bar{D}^2D_\gamma H^{(2)}_{\alpha(s)\dot\alpha(s)}+2B_2\Box H^{(2)}_{\alpha(s)\dot\alpha(s)} \\
&+\frac{2}{s!^2}B_3[D_{(\alpha_s},\bar{D}_{(\dot\alpha_s}][D^\gamma,\bar{D}^{\dot\gamma}]H^{(2)}_{\gamma\alpha(s-1))\dot\gamma\dot\alpha(s-1))} \\
&+\frac{2}{s!^2}B_4\partial_{(\alpha_s(\dot\alpha_s}\partial^{\gamma\dot\gamma}H^{(2)}_{\gamma\alpha(s-1))\dot\gamma\dot\alpha(s-1))}\quad .
\end{aligned} \tag{18}
$$

The gauge invariance of the action demands:

$$
\begin{aligned}
0=\delta\mathcal{S}=\int d^8z\Big\{&\delta\Psi^{\alpha(s)\dot\alpha(s-1)}\frac{\delta\mathcal{S}}{\delta\Psi^{\alpha(s)\dot\alpha(s-1)}}+\delta\bar\Psi^{\alpha(s-1)\dot\alpha(s)}\frac{\delta\mathcal{S}}{\delta\bar\Psi^{\alpha(s-1)\dot\alpha(s)}} \\
&+\delta H^{(1)\alpha(s-1)\dot\alpha(s-1)}\frac{\delta\mathcal{S}}{\delta H^{(1)\alpha(s-1)\dot\alpha(s-1)}} \\
&+\delta H^{(2)\alpha(s)\dot\alpha(s)}\frac{\delta\mathcal{S}}{\delta H^{(2)\alpha(s)\dot\alpha(s)}}\Big\} \\
=\int d^8z L^{\alpha(s)\dot\alpha(s-1)}&\Big\{\mathrm{D}^2\frac{\delta\mathcal{S}}{\delta\Psi^{\alpha(s)\dot\alpha(s-1)}}-\frac{1}{s!}\mathrm{D}_{(\alpha_s}\frac{\delta\mathcal{S}}{\delta H^{(1)\alpha(s-1))\dot\alpha(s-1)}}\Big\} \\
&+U^{\alpha(s)\dot\alpha(s-1)}\Big\{\bar{\mathrm{D}}^2\frac{\delta\mathcal{S}}{\delta\bar\Psi^{\alpha(s-1)\dot\alpha(s)}}+\bar{\mathrm{D}}^{\dot\alpha_s}\frac{\delta\mathcal{S}}{\delta H^{(2)\alpha(s)\dot\alpha(s)}}\Big\} \\
&+c.c.
\end{aligned} \tag{19}
$$

Therefore the two Bianchi identities for the on-shell fields strengths $\mathcal{T}_{\alpha(s)\dot\alpha(s-1)}$, $\mathcal{G}^{(1)}_{\alpha(s-1)\dot\alpha(s-1)}$, and $\mathcal{G}^{(2)}_{\alpha(s)\dot\alpha(s)}$ are given by:

$$
\mathrm{D}^2\mathcal{T}_{\alpha(s)\dot\alpha(s-1)}-\frac{1}{s!}\mathrm{D}_{(\alpha_s}\mathcal{G}^{(1)}_{\alpha(s-1))\dot\alpha(s-1)}=0 \tag{20a}
$$

$$
\bar{\mathrm{D}}^2\mathcal{T}_{\alpha(s)\dot\alpha(s-1)}+\bar{\mathrm{D}}^{\dot\alpha_s}\mathcal{G}^{(2)}_{\alpha(s)\dot\alpha(s)}=0 \tag{20b}
$$

and of course their complex conjugates.

The enforcement of the Bianchi identities, in other words the invariance of the full action under the gauge transformation, will fix the rest of the unconstrained parameters

Equation (20a) gives:

$$\begin{aligned}
0 = {} & (2A_1 - a_1)\,\frac{1}{s!}\mathrm{D}^2\bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_s}H^{(1)}_{\alpha(s-1))\dot\alpha(s-1)} \\
& - \frac{2}{s!}A_2\Box\mathrm{D}_{(\alpha_s}H^{(1)}_{\alpha(s-1))\dot\alpha(s-1)} \\
& - \frac{2}{s!(s-1)!}A_3\mathrm{D}_{(\alpha_s}[\mathrm{D}_{\alpha_{s-1}},\bar{\mathrm{D}}_{(\dot\alpha_{s-1}}][\mathrm{D}^\gamma,\bar{\mathrm{D}}^{\dot\gamma}]H^{(1)}_{\gamma\alpha(s-2))\dot\gamma\dot\alpha(s-2))} \\
& - \frac{2}{s!(s-1)!}A_4\mathrm{D}_{(\alpha_s}\partial_{\alpha_{s-1}(\dot\alpha_{s-1}}\partial^{\gamma\dot\gamma}H^{(1)}_{\gamma\alpha(s-2))\dot\gamma\dot\alpha(s-2))}
\end{aligned} \tag{21}$$

which fixes the coefficients, as promised, to the following values:

$$A_1 = \frac{1}{2}a_1 \tag{22a}$$
$$A_2 = 0 \tag{22b}$$
$$A_3 = 0 \tag{22c}$$
$$A_4 = 0 \quad . \tag{22d}$$

Equation (20b) gives:

$$\begin{aligned}
0 = {} & (a_2 - 2B_1)\,\bar{\mathrm{D}}^2\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha_s}H^{(2)}_{\alpha(s)\dot\alpha(s)} \\
& + 2B_2\Box\bar{\mathrm{D}}^{\dot\alpha_s}H^{(2)}_{\alpha(s)\dot\alpha(s)} \\
& + \frac{2}{s!^2}B_3\bar{\mathrm{D}}^{\dot\alpha_s}[\mathrm{D}_{(\alpha_s},\bar{\mathrm{D}}_{(\dot\alpha_s}][\mathrm{D}^\gamma,\bar{\mathrm{D}}^{\dot\gamma}]H^{(2)}_{\gamma\alpha(s-1))\dot\gamma\dot\alpha(s-1))} \\
& + \frac{2}{s!^2}B_4\bar{\mathrm{D}}^{\dot\alpha_s}\partial_{(\alpha_s(\dot\alpha_s}\partial^{\gamma\dot\gamma}H^{(2)}_{\gamma\alpha(s-1))\dot\gamma\dot\alpha(s-1))}
\end{aligned} \tag{23}$$

so we find that:

$$B_1 = \frac{1}{2}a_2 \tag{24a}$$
$$B_2 = 0 \tag{24b}$$
$$B_3 = 0 \tag{24c}$$
$$B_4 = 0 \quad . \tag{24d}$$

Hence the final action becomes

$$
\begin{aligned}
S_T = \int d^8 z \Big\{ & -\frac{s+1}{2s} a_2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \mathrm{D}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\
& -\frac{1}{2} a_1 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\
& + a_1 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}_{\alpha_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \\
& + a_2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \mathrm{D}_{\alpha_s} \bar{\mathrm{D}}^{\dot{\alpha}_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \\
& + a_2 \Big( \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} - \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \Big) H^{(2)}_{\alpha(s)\dot{\alpha}(s)} \\
& - a_1 \Big( \mathrm{D}_{\alpha_s} \bar{\mathrm{D}}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}_{\dot{\alpha}_s} \mathrm{D}^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \Big) H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} \\
& + \frac{1}{2} a_1 H^{(1)\alpha(s-1)\dot{\alpha}(s-1)} D^{\gamma} \bar{D}^2 D_{\gamma} H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} \\
& + \frac{1}{2} a_2 H^{(2)\alpha(s)\dot{\alpha}(s)} D^{\gamma} \bar{D}^2 D_{\gamma} H^{(2)}_{\alpha(s)\dot{\alpha}(s)} \Big\}
\end{aligned} \tag{25}
$$

and the superfield strengths take the form

$$
\begin{aligned}
\mathcal{T}_{\alpha(s)\dot{\alpha}(s-1)} = & -\frac{s+1}{s} a_2 \mathrm{D}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} - a_1 \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} \\
& + \frac{1}{s!} a_1 \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}_{(\alpha_s} \bar{\Psi}_{\alpha(s-1))\dot{\alpha}(s)} + \frac{1}{s!} a_2 \mathrm{D}_{(\alpha_s} \bar{\mathrm{D}}^{\dot{\alpha}_s} \bar{\Psi}_{\alpha(s-1))\dot{\alpha}(s)} \\
& + a_2 \mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}_s} H^{(2)}_{\alpha(s)\dot{\alpha}(s)} - \frac{1}{s!} a_1 \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_s} H^{(1)}_{\alpha(s-1))\dot{\alpha}(s-1)}
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
\bar{\mathcal{T}}_{\alpha(s-1)\dot{\alpha}(s)} = & -\frac{s+1}{s} a_2 \bar{\mathrm{D}}^2 \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} - a_1 \mathrm{D}^2 \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \\
& + \frac{1}{s!} a_1 \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}_{(\dot{\alpha}_s} \Psi_{\alpha(s)\dot{\alpha}(s-1))} + \frac{1}{s!} a_2 \bar{\mathrm{D}}_{(\dot{\alpha}_s} \mathrm{D}^{\alpha_s} \Psi_{\alpha(s)\dot{\alpha}(s-1))} \\
& - a_2 \bar{\mathrm{D}}^2 \mathrm{D}^{\alpha_s} H^{(2)}_{\alpha(s)\dot{\alpha}(s)} - \frac{1}{s!} a_1 \mathrm{D}^2 \bar{\mathrm{D}}_{(\dot{\alpha}_s} H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1))}
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
\mathcal{G}^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} = & + a_1 \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + a_1 \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}^2 \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \\
& + a_1 \mathrm{D}^{\gamma} \bar{\mathrm{D}}^2 \mathrm{D}_{\gamma} H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)}
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
\mathcal{G}^{(2)}_{\alpha(s)\dot{\alpha}(s)} = & + \frac{1}{s!} a_2 \bar{\mathrm{D}}_{(\dot{\alpha}_s} \mathrm{D}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1))} - \frac{1}{s!} a_2 \mathrm{D}_{(\alpha_s} \bar{\mathrm{D}}^2 \bar{\Psi}_{\alpha(s-1))\dot{\alpha}(s)} \\
& + a_2 \mathrm{D}^{\gamma} \bar{\mathrm{D}}^2 \mathrm{D}_{\gamma} H^{(2)}_{\alpha(s)\dot{\alpha}(s)}
\end{aligned} \tag{29}
$$

The form of these superfield strengths (and therefore the equations of motion on-shell) will change once we pick specific values for the parameters $a_1$ and $a_2$.

# 4 Distinguished Points in the Parameter Space $(a_1, a_2)$

Notice that our action has 2 unconstrained parameters $a_1, a_2$ (modulo scalings) and depending on their values the dynamics of the theory (equations of motion) change. Therefore we can say that the points in the parameter space $(a_1, a_2)$ represent different supersymmetric theories. Now we will focus on two very special points on this space, (2,0) and (0,2). Of course, due to the possibility of re-scaling the gauge parameter superfields, these distinguished points are the only physically meaningful models described by this unified treatment. We will prove that point (2,0) corresponds to the theory developed in [1] and describes a massless integer superspin supermultiplet and the point (0,2) corresponds to the theory developed in [2] and describes a massless half-integer superspin supermultiplet

## 4.1 Integer Superspin Action

Consider the case where $a_1 = 2$ and $a_2 = 0$, then we recover exactly the action that appears in [1][4]

$$\begin{aligned}\mathcal{S} = \int d^8z \Big\{ & - \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\ & + 2\Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}_{\alpha_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \\ & - 2\Big(\mathrm{D}_{\alpha_s} \bar{\mathrm{D}}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}_{\dot{\alpha}_s} \mathrm{D}^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)}\Big) H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} \\ & + H^{(1)\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}^\gamma \bar{\mathrm{D}}^2 D_\gamma H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} \Big\}\end{aligned} \tag{30}$$

By setting $a_2 = 0$ we make all the terms that are not invariant under the $\bar{\mathrm{D}}^2 U_{\alpha(s)\dot{\alpha}(s-1)}$ part of the transportation (4), vanish. That allow us to generalize a bit this piece of the transformation and still keeping the invariance of the action. This action is invariant under the, more general, transformations

$$\begin{aligned}\delta\Psi_{\alpha(s)\dot{\alpha}(s-1)} &= \mathrm{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{(s-1)!} \bar{\mathrm{D}}_{(\dot{\alpha}_{s-1}} \Lambda_{\alpha(s)\dot{\alpha}(s-2))} \\ \delta H^{(1)}_{\alpha(s-1)\dot{\alpha}(s-1)} &= \mathrm{D}^{\alpha_s} L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}^{\dot{\alpha}_s} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)}\end{aligned} \tag{31}$$

[4] with the constrained compensators expressed in terms of unconstraint prepotentials

The superfield strengths take the form:

$$
\begin{aligned}
\mathcal{T}_{\alpha(s)\dot\alpha(s-1)} =& -2\bar{\mathrm{D}}^2\Psi_{\alpha(s)\dot\alpha(s-1)} + \frac{2}{s!}\bar{\mathrm{D}}^{\dot\alpha_s}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1))\dot\alpha(s)} \\
& -\frac{2}{s!}\bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_s}H^{(1)}_{\alpha(s-1))\dot\alpha(s-1)}
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
\bar{\mathcal{T}}_{\alpha(s-1)\dot\alpha(s)} =& -2\mathrm{D}^2\bar\Psi_{\alpha(s-1)\dot\alpha(s)} + \frac{2}{s!}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}_{(\dot\alpha_s}\Psi_{\alpha(s)\dot\alpha(s-1))} \\
& -\frac{2}{s!}\mathrm{D}^2\bar{\mathrm{D}}_{(\dot\alpha_s}H^{(1)}_{\alpha(s-1)\dot\alpha(s-1))}
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
\mathcal{G}^{(1)}_{\alpha(s-1)\dot\alpha(s-1)} =& 2\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^2\Psi_{\alpha(s)\dot\alpha(s-1)} + 2\bar{\mathrm{D}}^{\dot\alpha_s}\mathrm{D}^2\bar\Psi_{\alpha(s-1)\dot\alpha(s)} \\
& + \frac{2}{s!}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_s}H^{(1)}_{\alpha(s-1))\dot\alpha(s-1)} \\
& - 2\frac{s-1}{s!}\mathrm{D}_{(\alpha_{s-1}}\bar{\mathrm{D}}^2\mathrm{D}^\gamma H^{(1)}_{\gamma\alpha(s-2))\dot\alpha(s-1)}
\end{aligned}
\tag{34}
$$

Based on (32) and (34) we can define:

$$
\begin{aligned}
\mathcal{I}_{\alpha(s-1)\dot\alpha(s-1)} =& \mathcal{G}^{(1)}_{\alpha(s-1)\dot\alpha(s-1)} + \mathrm{D}^{\alpha_s}\mathcal{T}_{\alpha(s)\dot\alpha(s-1)} \\
=& -2\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha_s}\bar\Psi_{\alpha(s-1)\dot\alpha(s)} + 2\frac{s-1}{s!}\mathrm{D}_{(\alpha_{s-1}}\bar{\mathrm{D}}^{\dot\alpha_s}\mathrm{D}^\gamma\bar\Psi_{\gamma\alpha(s-2))\dot\alpha(s)} \\
& -2\frac{s-1}{s!}\mathrm{D}_{(\alpha_{s-1}}\bar{\mathrm{D}}^2\mathrm{D}^\gamma H^{(1)}_{\gamma\alpha(s-2))\dot\alpha(s-1)}
\end{aligned}
\tag{35}
$$

and by applying to this a number of partial derivatives, contracting all the undotted indices and symmetrizing over all the dotted indices, after some algebra we find

$$
\begin{aligned}
&\partial^{\alpha_1}{}_{(\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_s}\mathcal{I}_{\alpha(s-1)\dot\alpha(s-1))} = \\
&= -\frac{2}{s}\mathrm{D}^2X_{\dot\alpha(2s-2)} \\
&\quad + 2\frac{s-1}{s}\partial^{\dot\beta\beta}\bar{\mathrm{D}}_{(\dot\alpha_{2s-2}}\mathrm{D}^2\partial^{\alpha_1}{}_{\dot\alpha_{2s-3}}\dots\partial^{\alpha_{s-2}}{}_{\dot\alpha_s}\bar\Psi_{\beta\alpha(s-2)\dot\beta\dot\alpha(s-1))} \\
&\quad + 2i\frac{s-1}{s}\partial^{\alpha_1}{}_{(\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-2}}{}_{\dot\alpha_{s+1}}\bar{\mathrm{D}}_{\dot\alpha_s}\mathrm{D}^2\bar{\mathrm{D}}^2\mathrm{D}^\beta H^{(1)}_{\beta\alpha(s-2)\dot\alpha(s-1))}
\end{aligned}
\tag{36}
$$

where

$$
\begin{aligned}
X_{\dot\alpha(2s-2)} =& \bar{\mathrm{D}}^{\dot\beta}\partial^{\alpha_1}{}_{(\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_s}\bar\Psi_{\alpha(s-1)\dot\beta\dot\alpha(s-1))} \\
& + (s-1)\partial^{\dot\beta\beta}\bar{\mathrm{D}}_{(\dot\alpha_{2s-2}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-3}}\dots\partial^{\alpha_{s-2}}{}_{\dot\alpha_s}\bar\Psi_{\beta\alpha(s-2)\dot\beta\dot\alpha(s-1))} \quad .
\end{aligned}
\tag{37}
$$

So the results in (36) implies:

$$
\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}\mathrm{D}^2X_{\dot\alpha(2s-2))} = -\frac{s}{2}(2s-2)!\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_s}\mathcal{I}_{\alpha(s-1)\dot\alpha(s-1))} \quad . \tag{38}
$$

and making use of the result in (33) we obtain:

$$\begin{aligned}
&\bar{\mathrm{D}}^2\partial^{\alpha_1}{}_{(\dot\alpha_{2s-1}}\dots\partial^{\alpha_{s-1}}{}_{\alpha_{s+1}}\bar{\mathcal{T}}_{\alpha(s-1)\dot\alpha(s))} = \\
&\quad = -2\bar{\mathrm{D}}^2\mathrm{D}^2\partial^{\alpha_1}{}_{(\dot\alpha_{2s-1}}\dots\partial^{\alpha_{s-1}}{}_{\alpha_{s+1}}\bar\Psi_{\alpha(s-1)\dot\alpha(s))} \\
&\qquad + 2i\partial^{\alpha_1}{}_{(\dot\alpha_{2s-1}}\dots\partial^{\alpha_{s-1}}{}_{\alpha_{s+1}}\partial^{\alpha_s}{}_{\dot\alpha_s}\Psi_{\alpha(s)\dot\alpha(s-1))} \\
&\qquad - 2\bar{\mathrm{D}}^2\mathrm{D}^2\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\alpha_s}H^{(1)}_{\alpha(s-1)\dot\alpha(s-1))} \quad .
\end{aligned} \tag{39}$$

Finally (32) yields:

$$\begin{aligned}
&\partial^{\alpha_1}{}_{(\dot\alpha_{2s-1}}\dots\partial^{\alpha_s}{}_{\dot\alpha_s}\mathcal{T}_{\alpha(s)\dot\alpha(s-1))} = \\
&\quad = -2\bar{\mathrm{D}}^2\partial^{\alpha_1}{}_{(\dot\alpha_{2s-1}}\dots\partial^{\alpha_s}{}_{\dot\alpha_s}\Psi_{\alpha(s)\dot\alpha(s-1))} \\
&\qquad + 2i\bar{\mathrm{D}}^{\dot\beta}\mathrm{D}^2\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_s}\bar\Psi_{\alpha(s-1)\dot\alpha(s-1))\dot\beta} \\
&\qquad - 2i\bar{\mathrm{D}}^2\mathrm{D}^2\partial^{\alpha_1}{}_{(\dot\alpha_{2s-1}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_{s+1}}\bar\Psi_{\alpha(s-1)\dot\alpha(s))} \\
&\qquad - 2i\bar{\mathrm{D}}^2\mathrm{D}^2\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_s}H^{(1)}_{\alpha(s-1)\dot\alpha(s-1))}
\end{aligned} \tag{40}$$

Based on (39) ,the identity:

$$\begin{aligned}
\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_s}\bar\Psi_{\alpha(s-1)\dot\alpha(s-1))\dot\beta} \;=& \\
&\left[\frac{1}{2s}\right]\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_s}\bar\Psi_{\alpha(s-1)\dot\alpha(s-1)\dot\beta)} \\
&+\left[\frac{2s-1}{(2s)!}\right]C_{\dot\beta(\dot\alpha_{2s-1}}X_{\dot\alpha(2s-2))}
\end{aligned} \tag{41}$$

and the definition

$$\bar{\mathcal{W}}_{\dot\alpha(2s)} = \mathrm{D}^2\bar{\mathrm{D}}_{(\dot\alpha_{2s}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-1}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_{s+1}}\bar\Psi_{\alpha(s-1)\dot\alpha(s))} \quad , \tag{42}$$

equation (40) becomes

$$\begin{aligned}
&\partial^{\alpha_1}{}_{\dot\alpha_{2s-1}}\dots\partial^{\alpha_s}{}_{\dot\alpha_s}\mathcal{T}_{\alpha(s)\dot\alpha(s-1))} = \\
&\quad = i\bar{\mathrm{D}}^2\partial^{\alpha_1}{}_{(\dot\alpha_{2s-1}}\dots\partial^{\alpha_{s-1}}{}_{\alpha_{s+1}}\bar{\mathcal{T}}_{\alpha(s-1)\dot\alpha(s))} \\
&\qquad + 2i\frac{2s-1}{(2s)!}\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}D^2X_{\dot\alpha_{2s-2})} \\
&\qquad + \frac{i}{s}\bar{\mathrm{D}}^{\dot\alpha_{2s}}\bar{\mathcal{W}}_{\dot\alpha(2s)}
\end{aligned} \tag{43}$$

so that with the help of (38) we obtain the relation between the physical field strength

superfield $\bar{\mathcal{W}}_{\dot\alpha(2s)}$ and the on-shell field strength superfields:

$$\begin{aligned}\bar{\mathrm{D}}^{\dot\alpha_{2s}}\bar{\mathcal{W}}_{\dot\alpha(2s)} =&\frac{s}{2}\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_s}\mathrm{D}^{\beta}\mathcal{T}_{\beta\alpha(s-1)\dot\alpha(s-1))}\\ &- is\partial^{\alpha_1}{}_{\dot\alpha_{2s-1}}\dots\partial^{\alpha_s}{}_{\dot\alpha_s}\mathcal{T}_{\alpha(s)\dot\alpha(s-1))}\\ &- s\bar{\mathrm{D}}^2\partial^{\alpha_1}{}_{(\dot\alpha_{2s-1}}\dots\partial^{\alpha_{s-1}}{}_{\alpha_{s+1}}\bar{\mathcal{T}}_{\alpha(s-1)\dot\alpha(s))}\\ &\frac{s}{2}\bar{\mathrm{D}}_{(\dot\alpha_{2s-1}}\partial^{\alpha_1}{}_{\dot\alpha_{2s-2}}\dots\partial^{\alpha_{s-1}}{}_{\dot\alpha_s}\mathcal{G}^{(1)}_{\alpha(s-1)\dot\alpha(s-1)}\quad .\end{aligned}\tag{44}$$

Also by its definition we find that:

$$\mathrm{D}_\alpha\bar{\mathcal{W}}_{\dot\alpha(2s)} = 0\tag{45}$$

Appropriately on-shell the field strength superfields indicated immediately below

$$\mathcal{T}_{\alpha(s)\dot\alpha(s-1)} = 0\tag{46a}$$

$$\mathcal{G}^{(1)}_{\alpha(s-1)\dot\alpha(s-1)} = 0\tag{46b}$$

vanish. Hence we find that

$$\bar{\mathrm{D}}^{\dot\alpha_{2s}}\bar{\mathcal{W}}_{\dot\alpha(2s)} = 0\ ,\qquad \mathrm{D}^{\alpha_{2s}}\mathcal{W}_{\alpha(2s)} = 0\tag{47a}$$

$$\mathrm{D}_\alpha\bar{\mathcal{W}}_{\dot\alpha(2s)} = 0\ ,\qquad \bar{\mathrm{D}}_{\dot\alpha}\mathcal{W}_{\alpha(2s)} = 0\tag{47b}$$

These are exactly the equations of motion needed [3, 6, 7] in order to describe an integer superspin $Y = s$ massless supermultiplet

## 4.2 Half-Odd Superspin Action

The case where $a_1 = 0$ and $a_2 = 2$, gives back the action proposed in [2]

$$\begin{aligned}\mathcal{S} = \int d^8z\Big\{ &-\frac{s+1}{s}\Psi^{\alpha(s)\dot\alpha(s-1)}\mathrm{D}^2\Psi_{\alpha(s)\dot\alpha(s-1)} + c.c.\\ &+ 2\Psi^{\alpha(s)\dot\alpha(s-1)}\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^{\dot\alpha_s}\bar\Psi_{\alpha(s-1)\dot\alpha(s)}\\ &+ 2\Big(\bar{\mathrm{D}}^{\dot\alpha_s}\mathrm{D}^2\Psi^{\alpha(s)\dot\alpha(s-1)} - \mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^2\bar\Psi^{\alpha(s-1)\dot\alpha(s)}\Big)H^{(2)}_{\alpha(s)\dot\alpha(s)}\\ &+ H^{(2)\alpha(s)\dot\alpha(s)}\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma H^{(2)}_{\alpha(s)\dot\alpha(s)}\Big\}\end{aligned}\tag{48}$$

By setting $a_1 = 0$, we make the terms which are not invariant under the $\mathrm{D}^2 L_{\alpha(s)\dot\alpha(s-1)}$ piece of (4) to vanish. So this piece of the transformation can be generalized without losing the invariance of the action. This action is invariant under the following transformations

$$\begin{aligned}\delta H^{(2)}_{\alpha(s)\dot\alpha(s)} &= \frac{1}{s!}\bar{\mathrm{D}}_{(\dot\alpha_s}U_{\alpha(s)\dot\alpha(s-1))} - \frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar U_{\alpha(s-1))\dot\alpha(s)}\\ \delta\Psi_{\alpha(s)\dot\alpha(s-1)} &= \bar{\mathrm{D}}^2 U_{\alpha(s)\dot\alpha(s-1)} + \mathrm{D}^{\alpha_{s+1}}\Lambda_{\alpha(s+1)\dot\alpha(s)}\end{aligned}\tag{49}$$

The superfield strengths are:

$$\begin{aligned}\mathcal{T}_{\alpha(s)\dot\alpha(s-1)} =& - 2\frac{s+1}{s}\mathrm{D}^2\Psi_{\alpha(s)\dot\alpha(s-1)} + 2\frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{\mathrm{D}}^{\dot\alpha_s}\bar\Psi_{\alpha(s-1))\dot\alpha(s)}\\ &+ 2\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha_s}H^{(2)}_{\alpha(s)\dot\alpha(s)}\end{aligned}\tag{50}$$

$$\begin{aligned}\bar{\mathcal{T}}_{\alpha(s-1)\dot\alpha(s)} =& - 2\frac{s+1}{s}\bar{\mathrm{D}}^2\bar\Psi_{\alpha(s-1)\dot\alpha(s)} + \frac{2}{s!}\bar{\mathrm{D}}_{(\dot\alpha_s}\mathrm{D}^{\alpha_s}\Psi_{\alpha(s)\dot\alpha(s-1))}\\ &- 2\bar{\mathrm{D}}^2\mathrm{D}^{\alpha_s}H^{(2)}_{\alpha(s)\dot\alpha(s)}\end{aligned}\tag{51}$$

$$\begin{aligned}\mathcal{G}^{(2)}_{\alpha(s)\dot\alpha(s)} =& + \frac{2}{s!}\bar{\mathrm{D}}_{(\dot\alpha_s}\mathrm{D}^2\Psi_{\alpha(s)\dot\alpha(s-1))} - \frac{2}{s!}\mathrm{D}_{(\alpha_s}\bar{\mathrm{D}}^2\bar\Psi_{\alpha(s-1))\dot\alpha(s)}\\ &+ 2\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma H^{(2)}_{\alpha(s)\dot\alpha(s)}\end{aligned}\tag{52}$$

From the above we find:

$$\begin{aligned}&\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+2}}{}^{\dot\alpha_{s-1}}\mathrm{D}_{\alpha_{s+1}}\bar{\mathrm{D}}^2\mathcal{T}_{\alpha(s))\dot\alpha(s-1)} =\\ &\qquad= -2\frac{s+1}{s}\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+2}}{}^{\dot\alpha_{s-1}}\mathrm{D}_{\alpha_{s+1}}\bar{\mathrm{D}}^2\mathrm{D}^2\Psi_{\alpha(s))\dot\alpha(s-1)}\\ &\qquad+ 2i\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+2}}{}^{\dot\alpha_{s-1}}\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^2\bar\Psi_{\alpha(s-1))\dot\alpha(s)}\\ &\qquad+ 2i\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+2}}{}^{\dot\alpha_{s-1}}\mathrm{D}_{\alpha_{s+1}}\bar{\mathrm{D}}^2\mathrm{D}^\beta\partial_\beta{}^{\dot\alpha_s}H^{(2)}_{\alpha(s))\dot\alpha(s)}\end{aligned}\tag{53}$$

$$\begin{aligned}&\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathrm{D}_{\alpha_s}\bar{\mathcal{T}}_{\alpha(s-1))\dot\alpha(s)} =\\ &\qquad= -2\frac{s+1}{s}\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^2\bar\Psi_{\alpha(s-1))\dot\alpha(s)}\\ &\qquad- 2i\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+2}}{}^{\dot\alpha_{s-1}}\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^2\mathrm{D}^2\Psi_{\alpha(s))\dot\alpha(s-1)}\\ &\qquad- 2\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^2\mathrm{D}^\beta H^{(2)}_{\beta\alpha(s-1))\dot\alpha(s)}\end{aligned}\tag{54}$$

and these two equations (53) and (54), combined give:

$$\begin{aligned}&\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+2}}{}^{\dot\alpha_{s-1}}\mathrm{D}_{\alpha_{s+1}}\bar{\mathrm{D}}^2\mathcal{T}_{\alpha(s))\dot\alpha(s-1)} - i\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathrm{D}_{\alpha_s}\bar{\mathcal{T}}_{\alpha(s-1))\dot\alpha(s)} =\\ &\quad- 2\frac{2s+1}{s}\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+2}}{}^{\dot\alpha_{s-1}}\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^2\mathrm{D}^2\Psi_{\alpha(s))\dot\alpha(s-1)}\\ &\quad+ 2i\frac{2s+1}{s}\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^2\bar\Psi_{\alpha(s-1))\dot\alpha(s)}\\ &\quad+ 2i\frac{2}{(2s)!}\mathrm{D}_{(\alpha_{2s}}\bar{\mathrm{D}}^2X_{\alpha(2s-1))}\end{aligned}\tag{55}$$

where

$$\begin{aligned}X_{\alpha(2s-1)} =& + s\partial_{(\alpha_{2s-1}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_{s-1}}\mathrm{D}^\beta\partial_\beta{}^{\dot\alpha_s}H_{\alpha(s))\dot\alpha(s)}\\ &+ s\partial_{(\alpha_{2s-1}}{}^{\dot\alpha_1}\dots\partial_{\alpha_s}{}^{\dot\alpha_s}\mathrm{D}^\beta H_{\beta\alpha(s-1))\dot\alpha(s)}\end{aligned}\tag{56}$$

From (52) we see:

$$
\begin{aligned}
\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathcal{G}^{(2)}_{\alpha(s))\dot\alpha(s)} = \\
&= -2i\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+2}}{}^{\dot\alpha_{s-1}}\mathrm{D}_{\alpha_{s+1}}\bar{\mathrm{D}}^2\mathrm{D}^2\Psi_{\alpha(s))\dot\alpha(s-1)} \\
&\quad - 2\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathrm{D}_{\alpha_s}\bar{\mathrm{D}}^2\bar\Psi_{\alpha(s-1))\dot\alpha(s)} \\
&\quad + \frac{2}{2s+1}\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_{(\gamma}\partial_{\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}H^{(2)}_{\alpha(s))\dot\alpha(s)} \\
&\quad - 2\frac{2s}{(2s+1)!}\mathrm{D}_{(\alpha_{2s}}\bar{\mathrm{D}}^2X_{\alpha(2s-1))}
\end{aligned}
\tag{57}
$$

due to the identity:

$$
\begin{aligned}
\mathrm{D}_\gamma\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}H^{(2)}_{\alpha(s))\dot\alpha(s)} =& \frac{1}{2s+1}\mathrm{D}_{(\gamma}\partial_{\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}H^{(2)}_{\alpha(s))\dot\alpha(s)} \\
&- \frac{2s}{(2s+1)!}C_{\gamma(\alpha_{2s}}X_{\alpha(2s-1))}
\end{aligned}
\tag{58}
$$

At this point we can define another chiral field strength superfield

$$
\mathcal{W}_{\alpha(2s+1)} = \bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_{2s+1}}\partial_{\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}H^{(2)}_{\alpha(s))\dot\alpha(s)} \quad . \tag{59}
$$

Finally equations (55), (57) and the adove definition when combined, give the following Bianchi identity

$$
\begin{aligned}
\mathrm{D}^{\alpha_{2s+1}}\mathcal{W}_{\alpha(2s+1)} =& \frac{2s+1}{2}\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathcal{G}^{(2)}_{\alpha(s))\dot\alpha(s)} \\
&- i\frac{s}{2}\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+2}}{}^{\dot\alpha_{s-1}}\mathrm{D}_{\alpha_{s+1}}\bar{\mathrm{D}}^2\mathcal{T}_{\alpha(s))\dot\alpha(s-1)} \\
&+ \frac{s}{2}\partial_{(\alpha_{2s}}{}^{\dot\alpha_1}\dots\partial_{\alpha_{s+1}}{}^{\dot\alpha_s}\mathrm{D}_{\alpha_s}\bar{\mathcal{T}}_{\alpha(s-1))\dot\alpha(s)}
\end{aligned}
\tag{60}
$$

and by definition

$$
\bar{\mathrm{D}}_{\dot\alpha}\mathcal{W}_{\alpha(2s+1)} = 0 \tag{61}
$$

On-Shell the superfield strengths vanish (equations of motion)

$$
\mathcal{T}_{\alpha(s)\dot\alpha(s-1)} = 0 \tag{62a}
$$

$$
\bar{\mathcal{T}}_{\alpha(s-1)\dot\alpha(s)} = 0 \tag{62b}
$$

$$
\mathcal{G}^{(2)}_{\alpha(s)\dot\alpha(s)} = 0 \quad . \tag{62c}
$$

Hence we get

$$
\mathrm{D}^{\alpha_{2s+1}}\mathcal{W}_{\alpha(2s+1)} = 0 \ , \qquad \bar{\mathrm{D}}^{\dot\alpha_{2s+1}}\bar{\mathcal{W}}_{\dot\alpha(2s+1)} = 0 \tag{63a}
$$

$$
\bar{\mathrm{D}}_{\dot\alpha}\mathcal{W}_{\alpha(2s+1)} = 0 \ , \qquad \mathrm{D}_{\alpha}\bar{\mathcal{W}}_{\dot\alpha(2s+1)} = 0 \quad . \tag{63b}
$$

This system describes a half odd superspin $Y = s + 1/2$ massless supermultiplet

## 5 Conclusion

In this work, we have presented a unified treatment of the work of Kuzenko et. al. Along the way, we have provided the first (to our knowledge) derivation of the explicit forms of the field strength superfields in terms of prepotentials and associated Bianchi identities. Our investigation also sets the stage for the study of possible alternative off-shell formulations of higher superspin superfield theories. The departure for this is to consider a generalization of the gauge transformation in (4) to include linear combinations of all the terms that appear in (3). This more general study will be undertaken in a later effort.

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